Lesson 4: Scaling Linear Sensors and Transducers

ET 438a

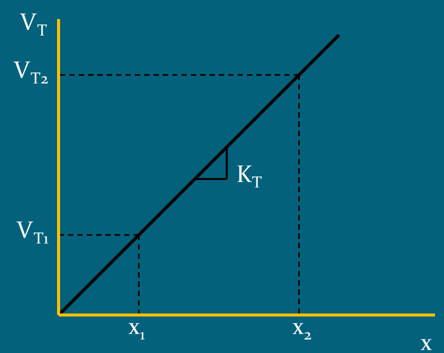
Automatic Control Systems Technology

Learning Objectives

After this presentation you will be able to:

- Develop mathematical relationships for a sensor that has a linear output.
- Convert linear mathematical equations into block diagrams that represent sensor scaling circuits.
- Adjust the output range of a sensor using operational amplifier circuits.

Scaling Sensor Outputs



$$K_T = \frac{\text{output span}}{\text{input span}} = \frac{S_o}{S_i}$$

x= transducer input (measured value)

V_T = transducer output voltage

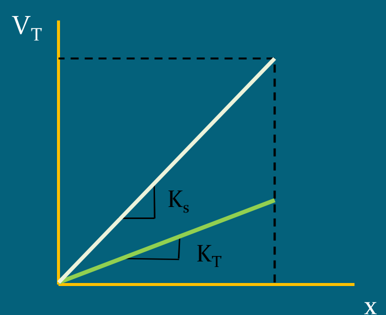
K_T = transducer gain (slope)

K_s = scalar gain

V_s = scalar output

Span = max. value - min. value

Sensor Scaling Case 1: No Offset in Scalar or Sensor



Transducer gain formula: $V_T = K_T x$

Required scalar gain:

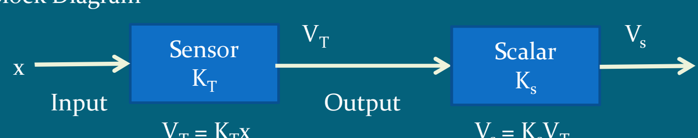
$$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$

Scalar output formula

$$V_s = K_s \cdot K_T \cdot x$$

$$V_s = K_s \cdot V_T$$

Block Diagram



Example 4-1: Case 1 Sensor

A pressure transducer has a usable range P (x) range of 0-50 psig (lb/in[2] gauge). It has a voltage output range of 0-1.25 Vdc over the pressure range. Scale the output to a range of 0-10 Vdc

Find K_T

$$K_T = \frac{\text{output span}}{\text{inputspan}} = \frac{S_o}{S_i} \qquad K_T = \frac{1.25 - 0\,V}{50 - 0\,\text{psig}} = 0.025\,V/\text{psig}$$

Find scalar gain

$$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T} \qquad K_s = \frac{10 - 0\ V}{1.25 - 0\ V} = 8\ V/V$$

P → [K_T=0.025 V/psig] → V_T → [K_s=8 V/V] → V_s

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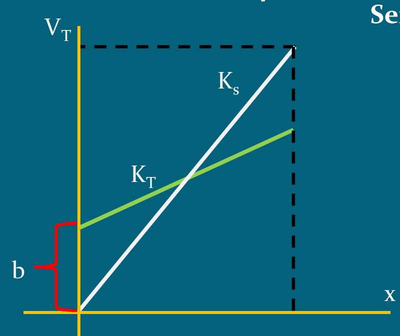
Sensor Scaling Case 2: Offset in Sensor No Offset in Output



Sensor gain formula: $V_T = K_T x + b$

Where b = transducer offset

Scalar gain formula must subtract offset

$$V_s = K_s(K_T x + b) - K_s b$$
$$V_s = K_s K_T x + K_s b - K_s b$$

Correct form of scalar output → $V_s = K_s K_T x$

Scalar equation → $V_s = K_s V_T - K_s b$

Block diagram

x → [Sensor $K_T \cdot x+b$] → V_T → [Scalar $K_s \cdot V_T - K_s \cdot b$] → V_s

$V_T = K_T \cdot x + b$ $\qquad$ $V_s = K_s \cdot V_T - K_s \cdot b$

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Example 4-2: Offset Sensor Output Range

A temperature transducer has a range of 0-100 C (input x) It has a voltage output range of 1-5 Vdc (V_T) over this temperature range. Scale the output to a range of 0-10 Vdc. Find the transducer gain, K_T, and the offset,b. Find the scalar relationship required to get the desired output range. Draw a block diagram of this sensor/scalar system that includes the mathematical relationship derived above.

Find K_T $$K_T = \frac{\text{output span}}{\text{input span}} = \frac{S_o}{S_i}$$ $$K_T = \frac{5-1\text{ V}}{100\text{ - }0\text{ C}} = 0.04\text{ V/C}$$

Find b from point slope form of line $$V_T - V_{T1} = K_T(x - x_1)$$

$$V_T - 1 = 0.04(x-0)$$
$$V_T = 0.04x + 1$$

b=1.0

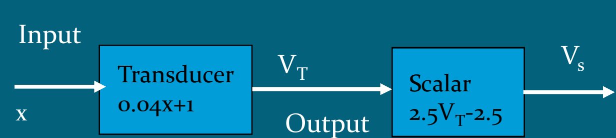
Example 4-2: Solution (2)

Find scalar gain $$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$ $$\frac{S_d}{S_T} = \frac{10-0\text{ V}}{5-1\text{ V}} = \frac{10}{4} = 2.5\text{ V/V} = K_s$$

$$V_s = K_s V_T - K_s b$$
$$V_s = 2.5V_T - 2.5(1)$$

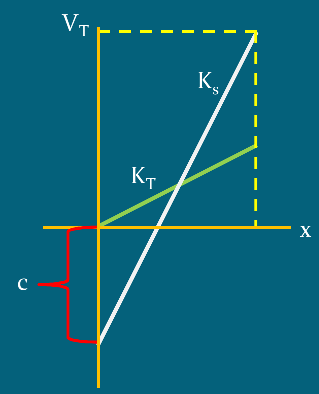
Scalar formula $$V_s = 2.5V_T - 2.5$$

Block Diagram



V_T= 0.04x+1 V_s = 2.5V_T-2.5

Sensor Scaling Case 3: Transducer with no offset, Output offset



Transducer gain formula:

$V_T = K_T x$

c = scalar offset can be +- value

Scalar formula must add a constant

$$V_s = K_s V_T + c$$

$$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$

To find c, use point slope form using scalar points

$$V_s - V_{s1} = K_s (V_T - V_{T1})$$

Example 4-3: Case 3 Sensor Scaling

A pressure transducer has an input range of 0 - 25 psig (x) and an output range of 0 - 1 V (V_T) Find the scaling equation to convert this range into the desired range of -5 V to +5 Vdc) Find transducer gain and scalar gain formulas. Draw the block diagram of the complete system

Find K_T $$K_T = \frac{\text{output span}}{\text{input span}} = \frac{S_o}{S_i}$$ $$K_T = \frac{1-0\ \text{V}}{25\ -\ 0\ \text{psig}} = 0.04\ \text{V/psig}$$

Find K_s $$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$ $$\frac{S_d}{S_T} = \frac{5-(-5)\ \text{V}}{1-0\ \text{V}} = \frac{10}{1} = 10\ \text{V/V} = K_s$$

Example 4-3: Solution (2)

Use point-slope form of line to find the value of c

$$V_s - V_{s1} = K_s(V_T - V_{T1})$$

$$(V_{T1}, V_{s1}) = (1, 5)$$

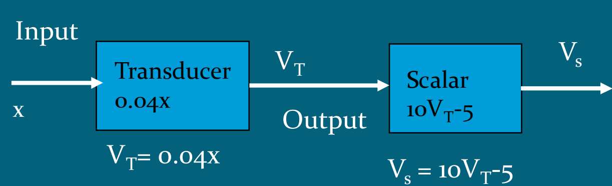
Can use either point in pair defining range

$$V_s - 5 = 10(V_T - 1)$$

$$V_s = 10V_T - 5$$

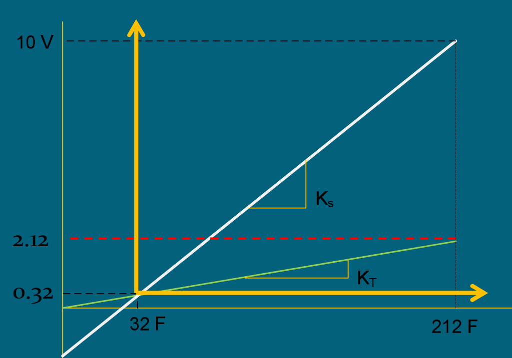
Scalar gain formula
c=-5

Block Diagram



Example 4-4 Scalar Equations

Range of linear temperature transducer is 32- 212 F with a transducer gain of 10 mV/F. The desired output of the transducer for the range of temperature is 0 - 10 Vdc. Find the gain formula.



Move origin to (32, 0.32)

Find span of transducer

At T_1 = 32 F
V_{T1} = 0.01 V/F (32 F) = 0.32 V

At T_2 = 212 F
V_{T2} = 0.01 V/F (212 F) = 2.12 V

span $S_T = V_{T2} - V_{T1}$
S_T = 2.12 − 0.32 V
S_T = 1.8 V

S_d = DESIRED SPAN
V_{max} = 10V V_{min} = 0
S_d = 10 - 0V = 10

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Example 4-4 Solution (2)

Compute sensor equation

$$V_T - 0.32 = \frac{2.12 - 0.32}{212 - 32}(T - 32)$$
$$V_T - 0.32 = \frac{1.8}{180}(T - 32)$$
$$V_T - 0.32 = 0.01(T - 32)$$
$$V_T = 0.01T - 0.32 + 0.32$$
$$V_T = 0.01T$$

Shift origin to (32 F, 0.32 V)
So b=0.32

$$V_S = K_S V_T - K_S b$$
$$K_S = \frac{S_d}{S_t} = \frac{10V}{1.8V} = 5.56 \text{ V/V}$$
$$V_S = 5.56 V_T - (5.56)(0.32) \text{ V/V}$$
$$V_S = 5.56 V_T - 1.779 \text{ V/V}$$

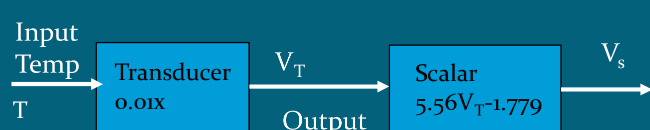
Check equation at data points

at 32 F $V_T = 0.32V$
$$V_S = 5.56(0.32V) - 1.779 = 0V$$

at 212 F $V_T = 2.12V$
$$V_S = 5.56(2.12) - 1.779 = 10.01V$$

Example 4-4 Solution (3)

Block Diagram

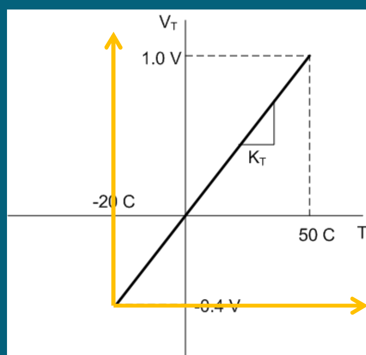


V_T= 0.01T

$V_s = 5.56V_T - 1.779$

Example 4-5

A linear temperature transducer has an input range of -20 C to 50 C and a gain of K_T = 20 mV/C. The desired output range is 0 - 5 Vdc. The transducer output voltage is bipolar (+-). Find the scaling equation.



Compute the transducer and scalar span

V_T at $-20°C$ $\quad V_{T1} = 20\,mV/C(-20C) = -0.4V$

V_T at $50°C$ $\quad V_{T2} = 20\,mV/C(50C) = 1.0V$

$S_T = 1.0 - (-0.4) = 1.4$

$S_S = 5 - 0 = 5$

Scalar formula

$V_S = K_S V_T - K_S b$

Find b graphically. Must shift origin to (-20 C, -0.4 V)

Example 4-5 Solution (2)

Find scalar gain

$K_S = \frac{S_d}{S_T} = \frac{5}{1.4} = 3.571\ V/V$

$V_S = 3.571V_T - 3.571(-0.4)$

$V_S = 3.571V_T + 1.429\ V/V$

Check scalar equation at data points

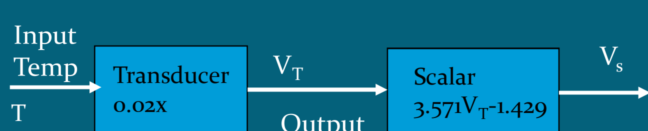
$V_T = -0.4V$ at $-20°C$

$V_S = 3.571(-0.4V) + 1.429 \approx 0V$

$V_T = 1.0V$ at $50°C$

$V_S = 3.571(1.0) + 1.429 = 5V$

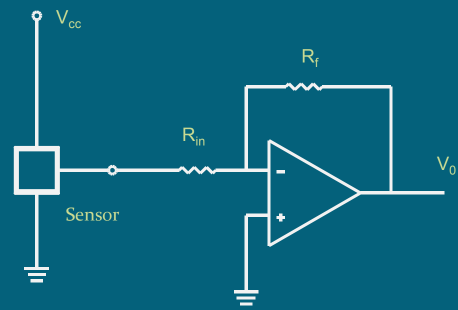
Block Diagram



V_T= 0.02T

$V_s = 3.571V_T - 1.429$

Practical Realization of Scalar Equations Using OP AMPs



For inverting amps

$$K_s = \frac{-R_f}{R_{in}}$$

For non-inverting amps

$$K_s = \left[1 + \frac{R_f}{R_1}\right]$$

Scaling without offset: use inverting or non-inverting amps to implement K_s

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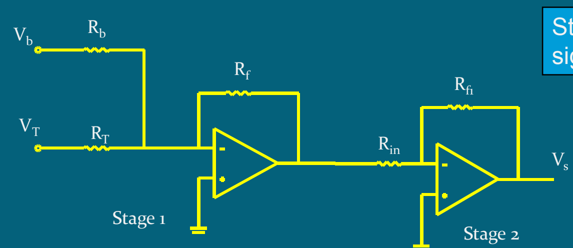
Practical Realization of Scalar Equations Using OP AMPs

For transducers with offset use inverting and summation amps



Stage 2 OP AMP changes sign if $R_{f1} = R_{in}$ $A_V = -1$

Stage 1

$$V_o = \frac{-R_f}{R_T} V_T + \frac{-R_f}{R_b} V_b$$

Overall gain

$$V_s = \left(\frac{-R_f}{R_T} V_T + \frac{-R_f}{R_b} V_b\right)(-1) = \frac{R_f}{R_T} V_T + \frac{R_f}{R_b} V_b$$

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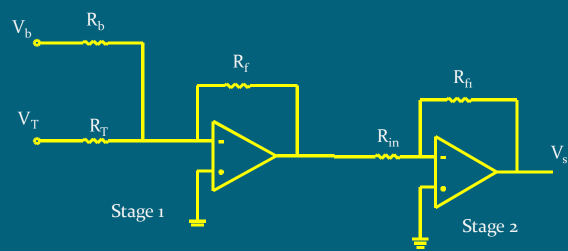
Practical Realization of Scalar Equations Using OP AMPs

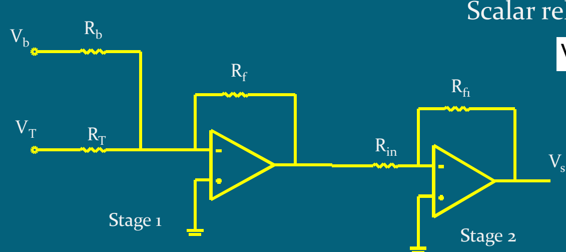


Equate the sensor with offset formula to the OP AMP gain derived on the last slide

$$V_s = (K_s V_T - K_s b) = \frac{R_f}{R_T} V_T + \frac{R_f}{R_b} V_b \quad \text{so} \quad K_s = \frac{R_f}{R_T} \quad K_s b = \frac{R_f}{R_b} V_b$$

Example 4-6: Implementing Scalar Equation with OP AMPs

Design an OP AMP circuit that will implement the scalar equation from Example 4-5 Assume $R_{fi} = R_{in} = 100\ k\Omega$ $R_f = 470\ k\Omega$



Scalar relationship for Example 4-5

$$V_s = 3.571 V_T + 1.429$$

$$K_s = \frac{R_f}{R_T}$$

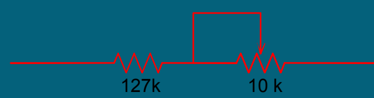
From scalar equation $K_s = 3.571$

Solve for R_T $\quad R_T = \frac{R_f}{K_s}$

$$R_T = \frac{470\,k\Omega}{3.571} = \boxed{131.61\,k\Omega}$$

Example 4-6: Solution (2)

Value of R_T is not a standard value. Use a standard value near the computed value in series with a potentiometer and calibrate circuit. Use -0.4 and 1.0 V and adjust the potentiometer



$$\frac{R_f}{R_1} = K_s \qquad \left(\frac{R_f}{R_b}\right)V_b = K_s b$$

R_f and $R_b > 0$ positive

Assume $V_b = 2.0V$ must be positive

$R_f = 470k\Omega$

from scalar equation

$K_s b = 1.429$

$$\left[\frac{470k\Omega}{R_b}\right](2.0V) = 1.429$$

$$(470k\Omega)(2.0) = 1.429 R_b$$

Example 4-6: Solution (3)

Compute the value of R_b

$$\frac{470k\Omega(2.0)}{1.429} = R_b$$

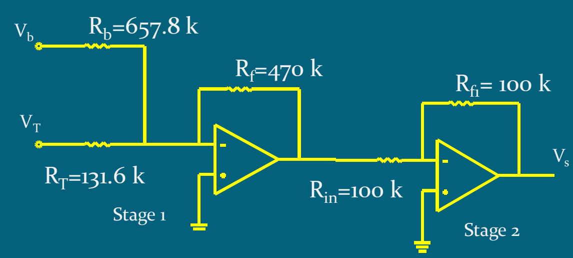
$$\boxed{657.8\,k\Omega = R_b}$$

Simpler design method: Let R_b=R_f

So.... $\frac{R_f}{R_b} = 1$, $K_s b = V_b$ Set V_b to the numerical value of $K_s b$

Example 4-6: Solution (4)

Completed design



End Lesson 4: Scaling Linear Sensors and Transducers

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